Kuwait University | Math 102 | October 24, 1996

Dept of Math. and Comp. Sc. | First Examination | Duration: 75 minutes

Calculators and mobile phones are not allowed.

Answer all of the following questions.

1. Find $\frac{dy}{dx}$ if

(a) $y^2 + \ln|y \cdot 4| + xy + \tanh x = 0,$

(b) $y = \dfrac{(x + \ln x)(x^5 + 7x)^{10}}{\sqrt{x+11}}.$

4 points each

2. Rewrite $\cos(\arctan(3x))$ as an algebraic expression of x if $x > 0$.

3 points

3. (a) Prove that the function f defined by

$$f(x) = 1 + 2e^{\sqrt{x}} \qquad (x \geq 0)$$

is one-to-one and find its inverse f^{-1}.

(b) State the domain and the range of f^{-1}

4+1 points

4. Evaluate the following integrals

(a) $\displaystyle\int \left(\frac{1}{\sec(5x)} + \sec(2-3x) \right) dx,$ (b) $\displaystyle\int \frac{dx}{x((\log_3 x)^2 + 1)}$

(c) $\displaystyle\int \frac{dx}{(\sec^{-1} x)x\sqrt{x^2-1}},$ (d) $\displaystyle\int \frac{\left(2^{x+1} - 2^{3x}\right)^2}{2^5} dx.$

4 points each

5. Find the limits

$$\lim_{x \to 0} \frac{\arctan x^2}{\sqrt{x^2+1} - 1}, \qquad \lim_{x \to -\infty} \frac{\ln \sqrt{x+10}}{\ln \sqrt{2x+4}}.$$

4 points each

Total 40 points